COMPARISON BETWEEN STATE SPACE AVERAGING AND PWM SWITCH FOR SWITCH MODE POWER SUPPLY ANALYSIS

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ABSTRACT

The state space averaging method has been very fruitful in determining DC and small signal analysis of switching mode power supplies. The method begins by writing the state equations and the state matrices describing the system, and requires matrix operations for computation of the system transfer functions. It also leads to a canonical model that represent the four basic dc-to dc topologies. On the other hand the newly introduced method of the ***PWM Switch*** *leads to a simpler circuit analysis of the converters which can be easily simulated using SPICE like simulation programs. A review of both methods is presented with graphical results for comparison.*

1.0 Introduction

The *state space averaging* technique [1] has been a useful tool which represents a simple, yet accurate, model for the DC and the small signal analysis of PWM converters, from which frequency response transfer functions of the converter; such as output impedance or audiosucepetibility; can be obtained. An overview of the state space averaging technique is presented in section 2, where these small signal transfer functions are deduced from the state space averaging representation of the system. In section 3 an introduction to the *PWM Switch* modeling technique is presented, followed in section 4 by a comparison between both modeling concepts, where two transfer functions of a buck converter operating in the continuous conduction mode obtained from both methods are plotted.

2.0 State Space Averaging

The state space averaging technique represents, in its final form, the small signal behavior of switching converters in terms of a set of linear time invariant state equations driven by a continuous duty ratio modulation function. The assumptions for the validity of the state space averaging is that the natural time constants of the converter are much longer than the switching period of operation; in other words, the natural frequencies of the converter (poles) are much lower than the switching frequency. A consequence of this assumption, is that the system can be transformed into a continuous time system with very small errors in the transfer functions noticeable at frequencies close to and above one half the switching frequency. This allows well known analysis tools, such as *Laplace transforms* and *Bode* plots, to be used for solving system transfer functions.

The formulation of this technique begins by writing down the differential equations that describe the linear circuit during the ON interval (T_{ON}) and the OFF interval (T_{OFF}) of the switching period (T_s). This operation can be described by the following equations:

$$\dot{x}(t) = A_i(t)x(t) + B_i(t)u(t), \qquad (1-a)$$

and

$$y(t) = C_i(t)x(t) + E_i(t)u(t) \qquad (1-b)$$

where i = 1 during dT_s and i = 2 during $d'T_s$, where d is the duty ratio, defined as T_{ON}/T_s, d' its complement. X(t) is the state vector which consists usually of the inductor current and the capacitor voltage, u(t) is the input vector which consists of all independent voltage and current sources in the converter, y(t) is the output vector, A(t) is the system matrix, B(t) is the input matrix, C(t) is the output matrix and E(t) is the transmission matrix.

The averaging process combines the four equations derived in eq.(1) into two described as:

$$\dot{x} = A(t)x(t) + B(t)u(t) \qquad (2)$$

$$y(t) = C(t)x(t) + E(t)u(t) \qquad (3)$$

where A , B, C and E are the average matrices of the system over the switching period computed, for example, as:

$$A = A_1 d + A_2 d' \qquad (4)$$

with similar expressions for B, C and D matrices.

Introducing a small signal perturbation given by:

$$x = X + \hat{x} \qquad (5-a)$$

$$y = Y + \hat{y} \qquad (5-b)$$

$$d = D + \hat{d} \qquad (5-c)$$

where the capital case letter represents a DC quantity and the lower case letter with a hat represents a small perturbation superimposed on the DC quantity. Substituting these quantities in (2) and (3), then performing the algebra and linearizing by ignoring products of small signal terms will result in DC solutions given by:

$$X = -A^{-1}BU \qquad (6)$$

and

$$Y = CX + EU \qquad (7)$$

and small signal solutions given by:

$$\dot{\hat{x}} = A\hat{x} + B\hat{u} + k\hat{d} \qquad (8)$$

and

$$\hat{y} = C\hat{x} + E\hat{u} + l\hat{d} \qquad (9)$$

where

$$k = (A_1 - A_2)X + (B_1 - B_2)U \qquad (10-a)$$

and

$$l = (C_1 - C_2)X + (E_1 - E_2)U \qquad (10-b)$$

From the above equations various transfer functions can be derived such as:

$$\frac{\hat{y}}{\hat{d}} = C(sI - A)^{-1}k + (C_1 - C_2)X \qquad (11)$$

Note that the k and l matrices depend on the steady state operating points, X and U.

(13)

3.0 PWM SWITCH

The ***PWM switch*** model has been introduced recently [2]. This model leads to considerable simplification in the analysis of the non-linear dc-to-dc converters, by which DC and small-signal AC analysis can be performed by simple circuit analysis.

Figure 1 shows a buck converter with the *PWM Switch* designated in the square block composed of the the transistor and the diode. The dc and ac model of the PWM switch derived in [2] are shown in figure 2.

3.1 DC Analysis:

To compute the dc conversion ratio of the buck converter, the dc model of the PWM switch is used as shown in figure 2, with the small signal generators and reactive elements are either shorted or opened. From the figure the dc conversion ratio M can be computed as:

$$M = \frac{V_o}{V_g} = D\frac{R}{R_L + R} \tag{12}$$

3.2 AC Analysis:

The AC model of the PWM switch in the buck converter is shown in figure 2, the relevant transfer functions can be computed as follows:

3.2.2 Duty Ratio to Output Transfer Function

The computation of the duty ratio to output transfer function follows from figure 2, shorting out $\hat{v}_g$ in the ac model and computing the $\hat{v}_o/\hat{d}$. Note that the dc operating point V_{ap} and I_c are V_g and I_o respectively, hence:

$$\frac{\hat{v}_o}{\hat{d}} = V_g\frac{(1 + sCR_c)}{\Delta}$$

where Δ is defined as:

$$\Delta = s^2[LC(R_c + R)] + s[L + CR_L(R_c + R) + CR_cR] + R \tag{14}$$

3.2.3 Output Impedance

This can be found by injecting an external voltage source and computing the voltage to current ratio to obtain:

$$Z_o = \frac{R[s^2(LCR_c) + s(CR_c + L) + R_L]}{\Delta} \tag{15}$$

where Δ as defined in (14).

4.0 Comparison

The system matrices are derived for the state space averaging method, and a computer program is written to extract the relevant transfer functions. These transfer functions are plotted in Figures 3 and 4 with those derived using the *PWM switch* model from (13) and (15). As seen from the plots, both the state space averaging and the PWM switch technique, give exactly the same results. The plots of both methods coincide, with a slight difference in the output impedance plot at low frequencies, this discrepancy is attributed to computational error in the evaluation of the matrices of the state space averaging method. As can be noted form the above discussion, the main difference between the state space averaging and the PWM switch method, is that in the former method, the state equations describing the *total* system are linearized, while in the *PWM Switch* method only the non-linear part responsible for the switching is linearized. Both techniques use averaging of the quantities of interest, hence they are both accurate for frequencies well below half of the switching frequency.

5.0 Conclusion

Both the state space averaging method and the *PWM Switch* method for modeling of dc-to-dc converters were reviewed. A comparison between both methods was demonstrated through the computation and plotting of typical transfer functions of a buck converter. Both methods simplify considerably the analysis of the non-linear converter system.

Both the state space averaging and the *PWM switch* techniques are applicable for ideal switches. The authors are currently conducting research on modeling PWM converters to include the non-ideal properties of the switches, such as rise time, fall time etc. The new model is based on the *PWM switch* concept with the introduction of a switching function to account for those non-idealities of the semicondcutor switches.

6.0 References

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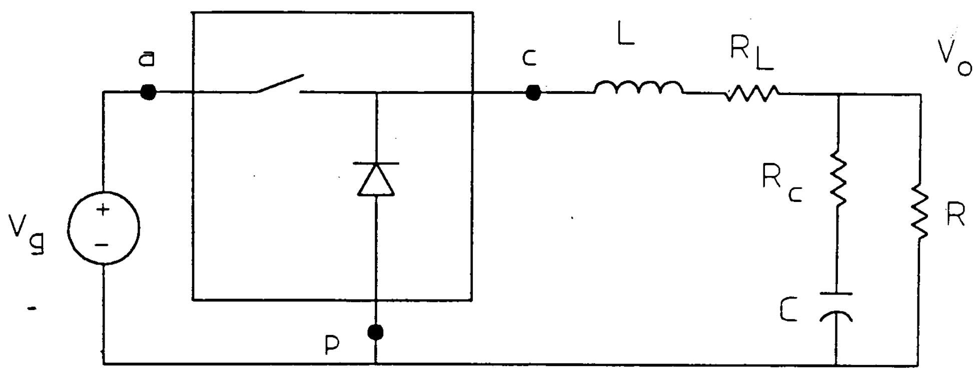

Figure 1: Buck Converter

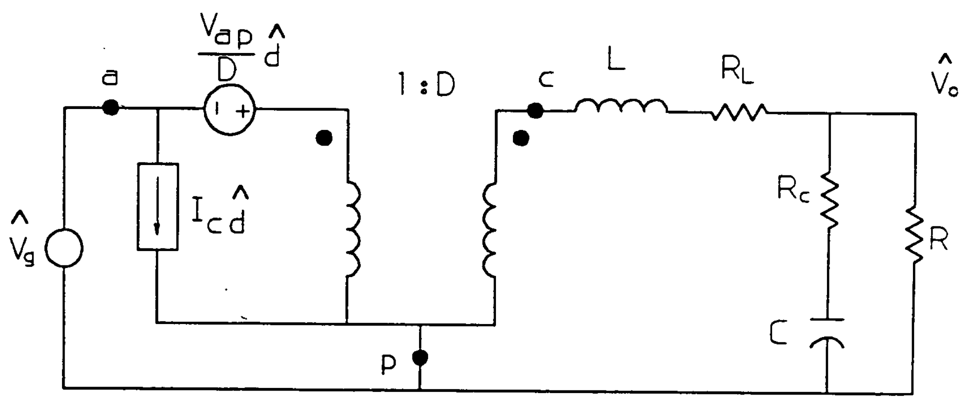

Figure 2: Buck Converter Model

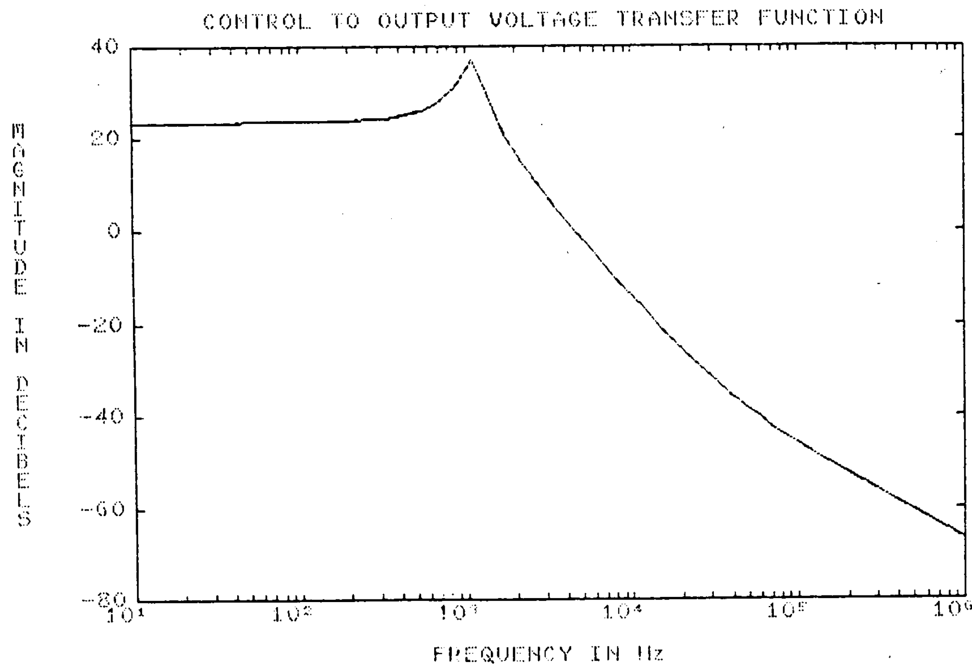

Figure 3: Control to Output Transfer Function

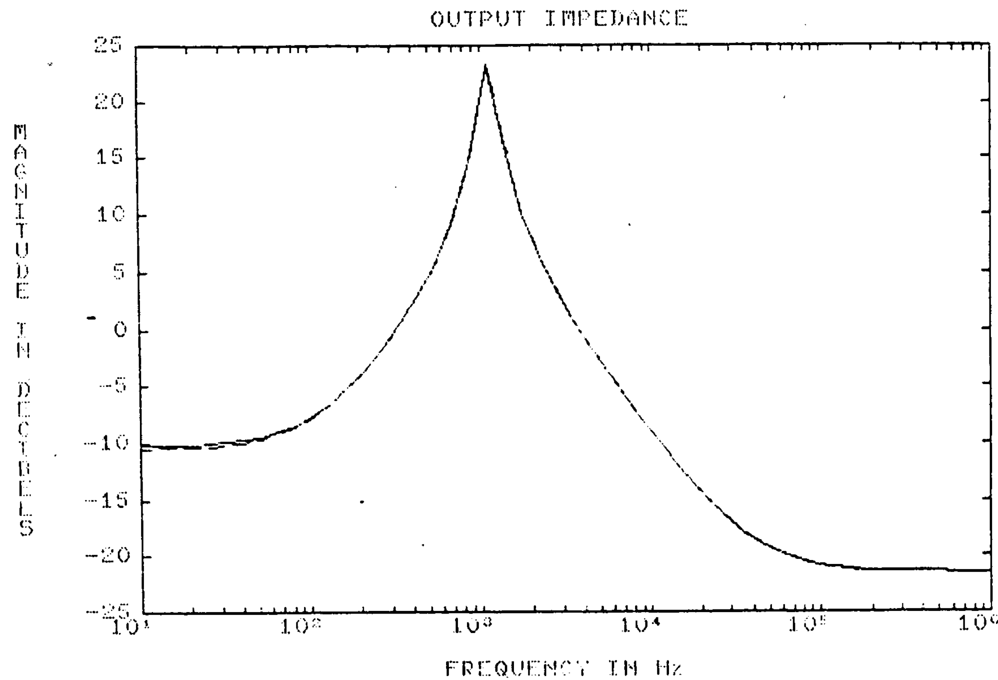

Figure 4: Output Impedance Transfer Function