An Overview on Repetitive Control --- what are the issues and where does it lead to?

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Agenda

- What is repetitive control?
- Its historical background
- What are the issues/difficulties?
- Theoretical Problems
- ILC (Iterative Learning Control) and Repetitive Control
- Future issues

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Selected References

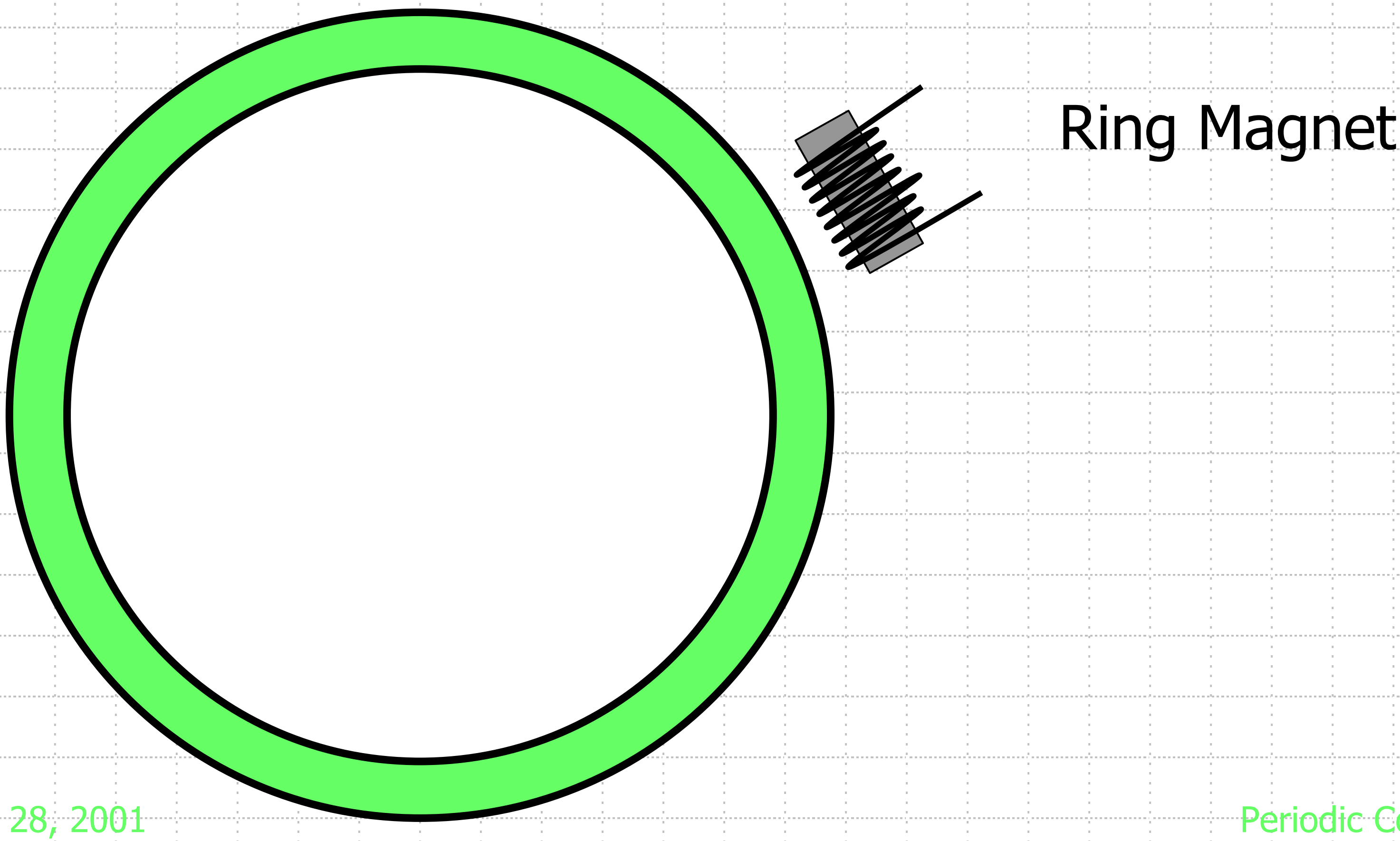
- Inoue, Nakano, etc.: IFAC Congress '81; Introduction of the idea
- Hara, Yamamoto et al.: IEEE Trans. '88, Stability analysis, design methods
- Yamamoto and Hara: IEEE Trans. AC, '88; Internal model principle in a general context
- Hara et al.: Proc. 29th CDC '90, digital repetitive control
- Yamamoto: 2nd ECC (Groningen) '93 "Perspectives in Control" (Birkhauser); Survey from the viewpoint of ∞-dim. system theory

What does it do?: Examples

- Repetitive control intends to track/reject arbitrary periodic signals of a fixed period
- Tracking/Disturbance rejection of periodic signals appear in many applications
  - Hard disk/CD drives
  - Electric power supply
  - Robotic motions
  - Steppers in IC productions
  - And many others

History: The First Example

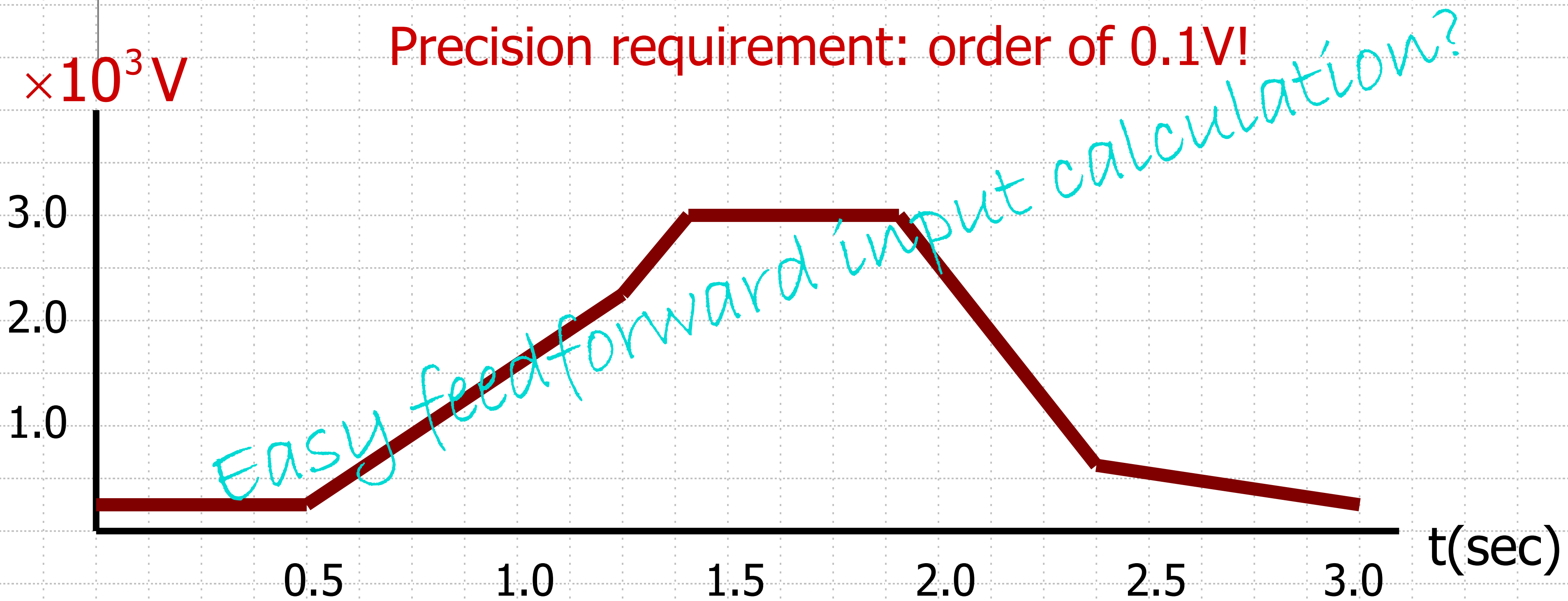
- Magnet power supply for a proton synchrotron (Nakano and others)



Control Objective:

- Control the power supply curve (periodically) to the following shape:

Precision requirement: order of 0.1V!



Difficulties

- Difficult to attain 10E-4 precision by computing the inverse system
  - Identification was difficult up to this precision
- Robustness requirement (robust tracking against plant uncertainty)
- Thus the 1st trial failed.
- *What to do?*

Solution (Nakano et al.'81)

- The reference signal is *periodic.*
- *Make the system learn the desired input by itself.*
  - Feed the reference into the plant;
  - Store the error signal for 1 period;
  - Then feed the error back into the plant, and so on.
- If we are lucky, we are in business.

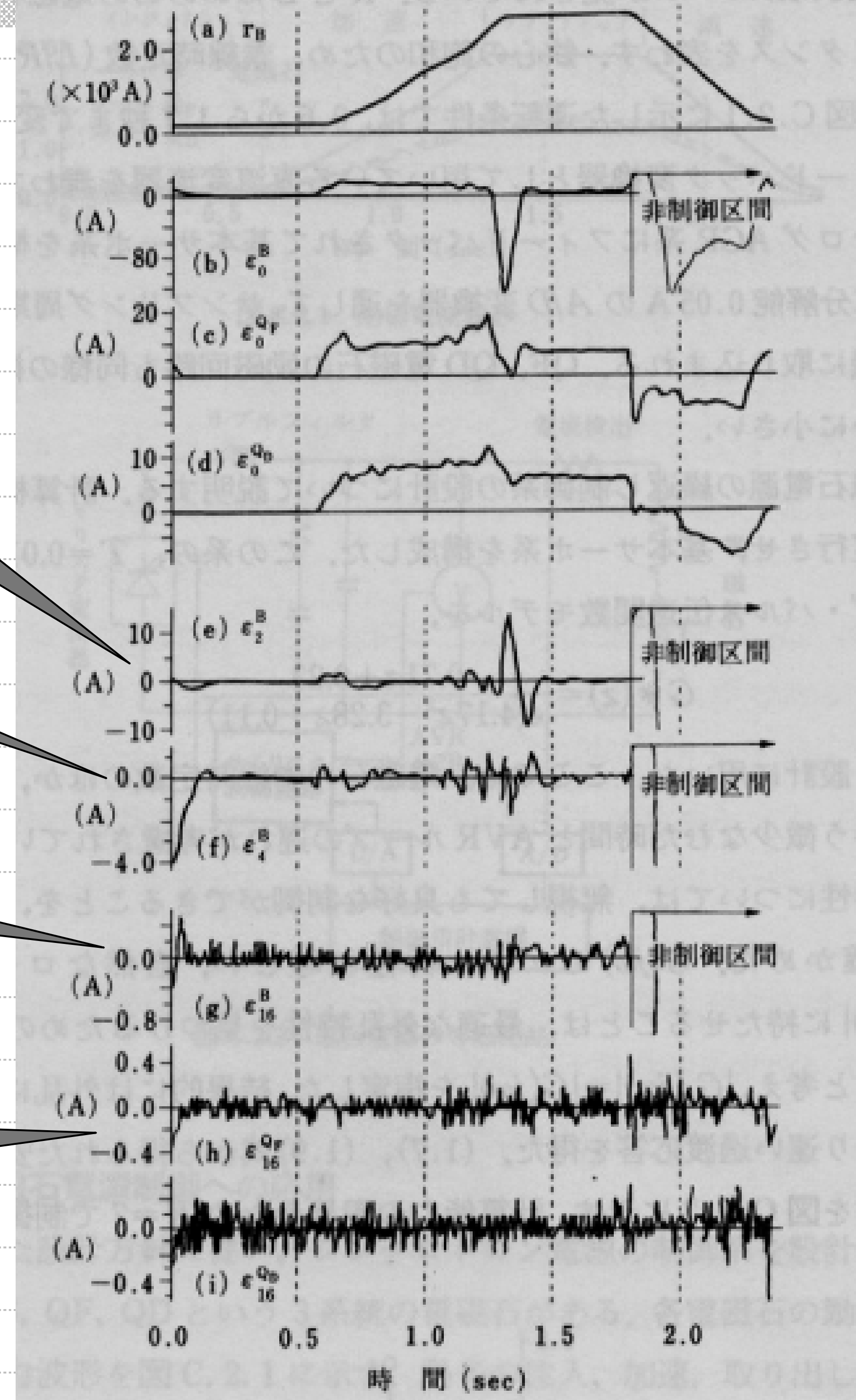
Result

Error in 2nd iteration

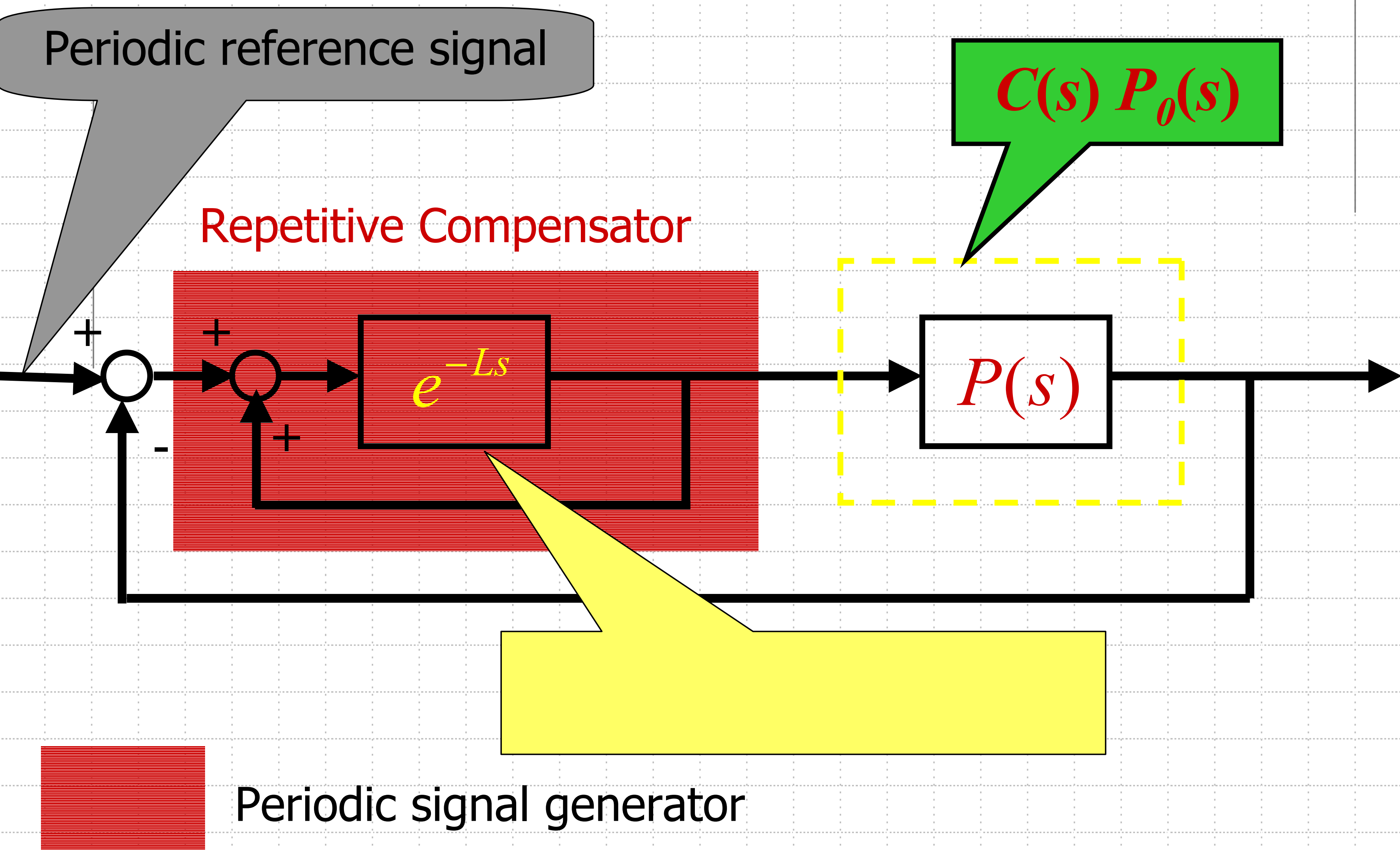
Error in 4th iteration

After 16th iteration

Another magnet



The General Construction



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- **What are the issues/difficulties?**
- Theoretical Problems
- Relationships with ILC
- Future issues

Basic Questions:

- What does this system amount to?
- Is the construct mandatory?
- Stability condition
  - Easy to stabilize?
- If not easy to stabilize, what to do?

What does this amount to?

- Servomechanism control system with periodic signal generator
- Attempts to track any periodic signal of a fixed period L
- Repetitive compensator works as an internal model for periodic signals

Is this construct mandatory?

- If we want to track any periodic signal, yes.
- What is the "minimal" system that generates all periodic signals of period L?
- Minimal realization = $1/(\exp(Ls) - 1)$
  - In what sense is this minimal?
  - How can this be shown?

Minimality of 1/(exp(Ls)-1)

Fourier series expansion $r(t) = \sum_{n=-\infty}^{\infty} r_n \exp(2n\pi jt/L)$

⇒ Need poles at 2nπj/L

(Internal Model Principle)

⇒ q(s) := sΠ (s - 2nπj/L) ?

This product diverges

Need Hadamard factorization

⇒ q(s) = sΠ (1 – Ls/2nπj)

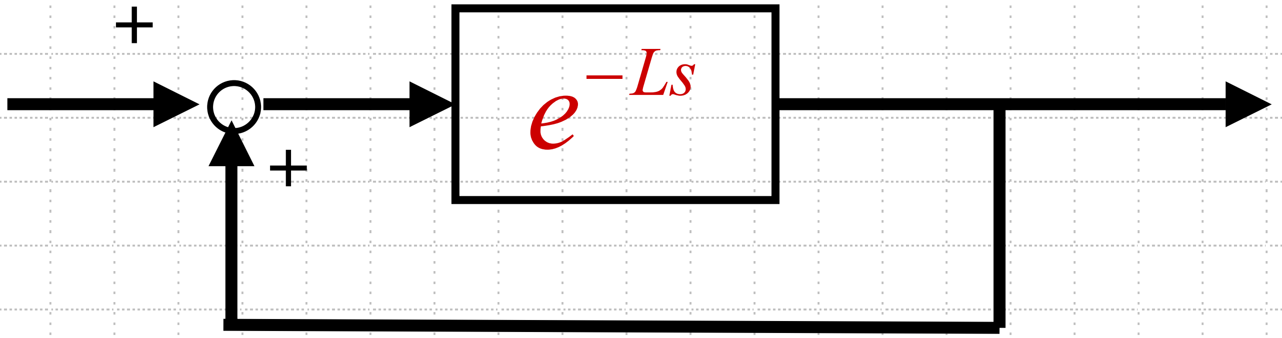
$$q(s) = s\prod_{n=1}^{\infty}(1 + L^2 s^2 / 4n^2\pi^2)$$

Note $\sinh \pi s = \pi s \prod_{n=1}^{\infty} (1 + s^2 / n^2)$

$$q(s) = \frac{2\pi}{\exp(Ls/2) - \exp(-Ls/2)}$$

$$q(s) = \frac{1}{\exp(Ls) - 1}$$

is the minimal model that contains all poles $2n\pi j/L$



Repetitive compensator

Skip PSR

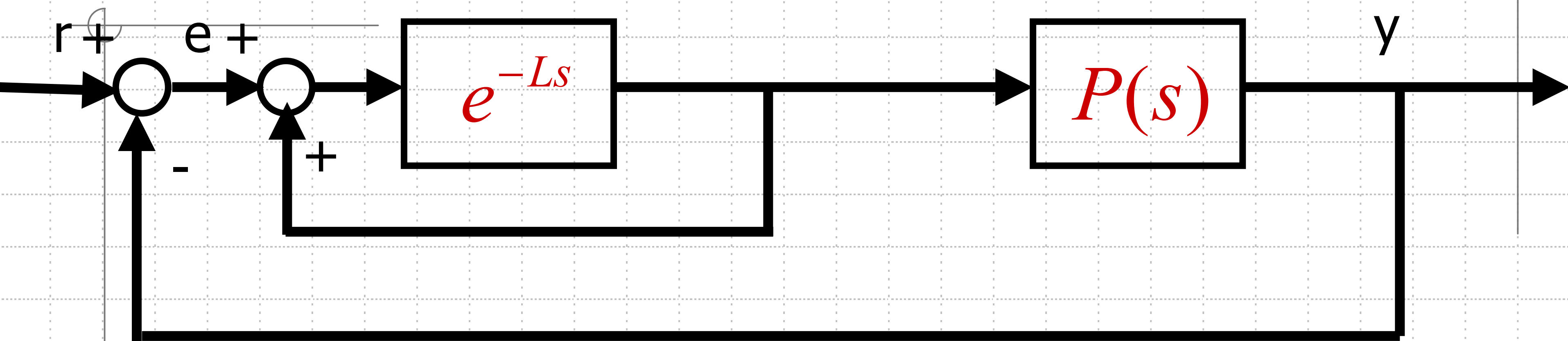
The Role of Repetitive Compensator

- Generates any (locally L^2) periodic signal of period L with suitable initial function stored in the delay
- Works as the internal model for periodic signals
- In what sense is this *minimal*?
- Not trivial in a more general context due to infinitely many unstable poles

An Infinite-dim. Representation Framework

- -- Yamamoto (SIAM'88; TAC'88)
- Pseudorational impulse responses
- $y = \pi(q^{-1} * p * u)$, $q, p \in \mathcal{E}'$
- $q \in \mathcal{E}' \Leftrightarrow \hat{q}(s)$ satisfies i) entire, ii) the Paley-Wiener estimate
  - $|\hat{q}(s)| \leqq (1+|s|)^m \exp(a|\mathrm{Re}\, s|)$
- d is contained as an internal model in q ⇔ d|q ⇔ $q=d*r$ in $\mathcal{E}'$ ⇔ q/d in the Paley-Wiener class
- ⇒ General internal model principle

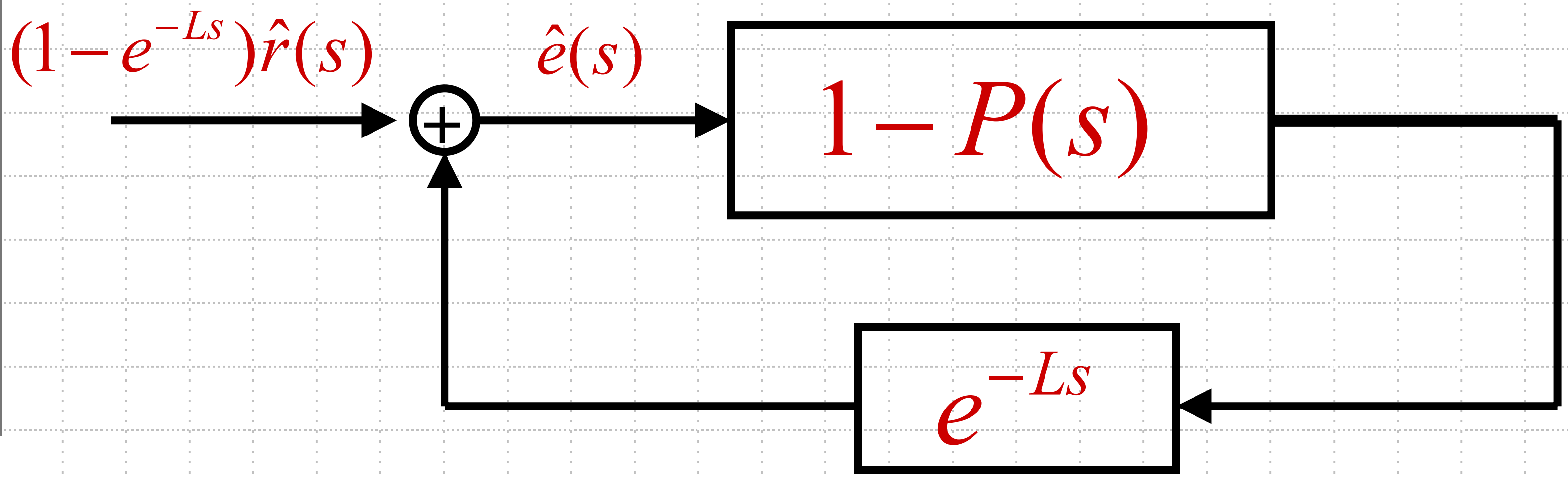
Tracking and Stability Conditions



$$W_{er}(s) = \frac{1}{1+P(s)/(e^{Ls}-1)} = \frac{e^{Ls}-1}{e^{Ls}-1+P(s)}$$

- S= $2n\pi j/L$, n=0, ±1, ±2, … are transmission zeros of $W_{er}(s)$.
- $r(t) = \sin(2n\pi jt/L)$ becomes unobservable
- Tracking accomplished under closed-loop stability

Closed-loop Stability

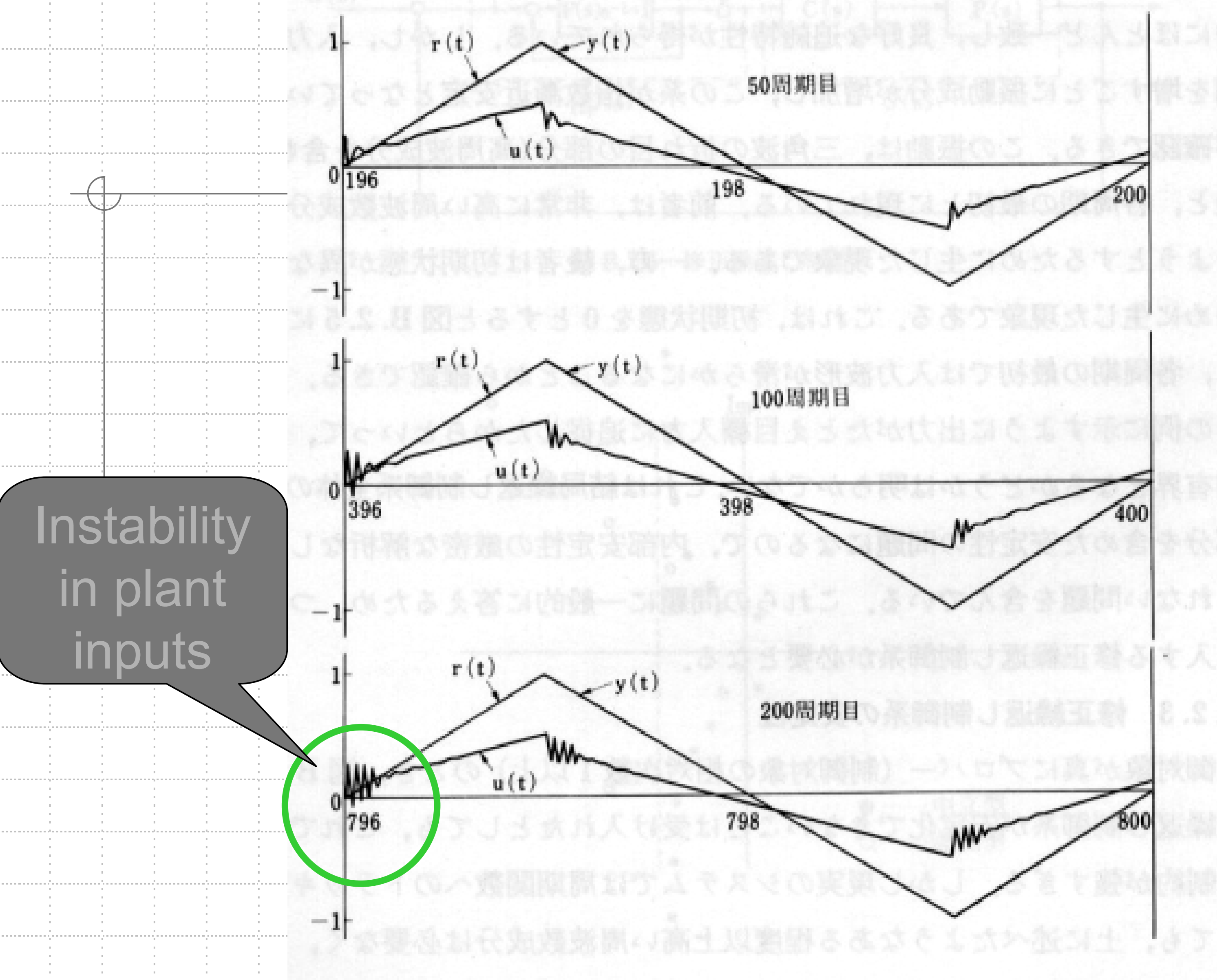


Equivalent diagram for $W_{er}(s)$

$$\|1-P(s)\|_\infty < 1 \Rightarrow L^2-\text{stability}$$

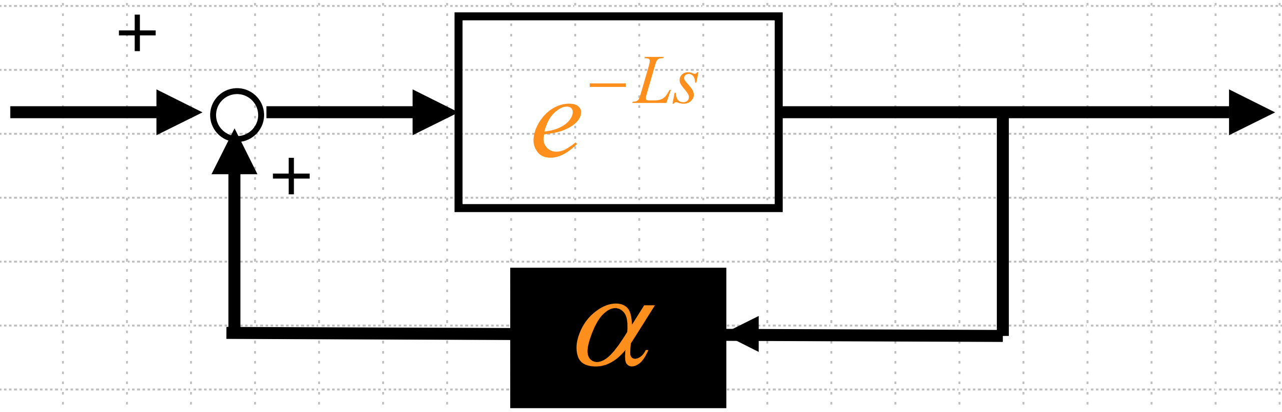
Problems in Stabilizability

- Condition close to necessity
- But almost never satisfied unless P is biproper ($P(\infty) \neq 0$)
- The stability is almost entirely governed by the feedback loop of the repetitive compensator
- Requires too much: tracking to arbitrary periodic signals (even discont. ones)



Response of a repetitive control system that is not Internally stable $P(s) = (15s+15)/(2s^2+20s+15)$

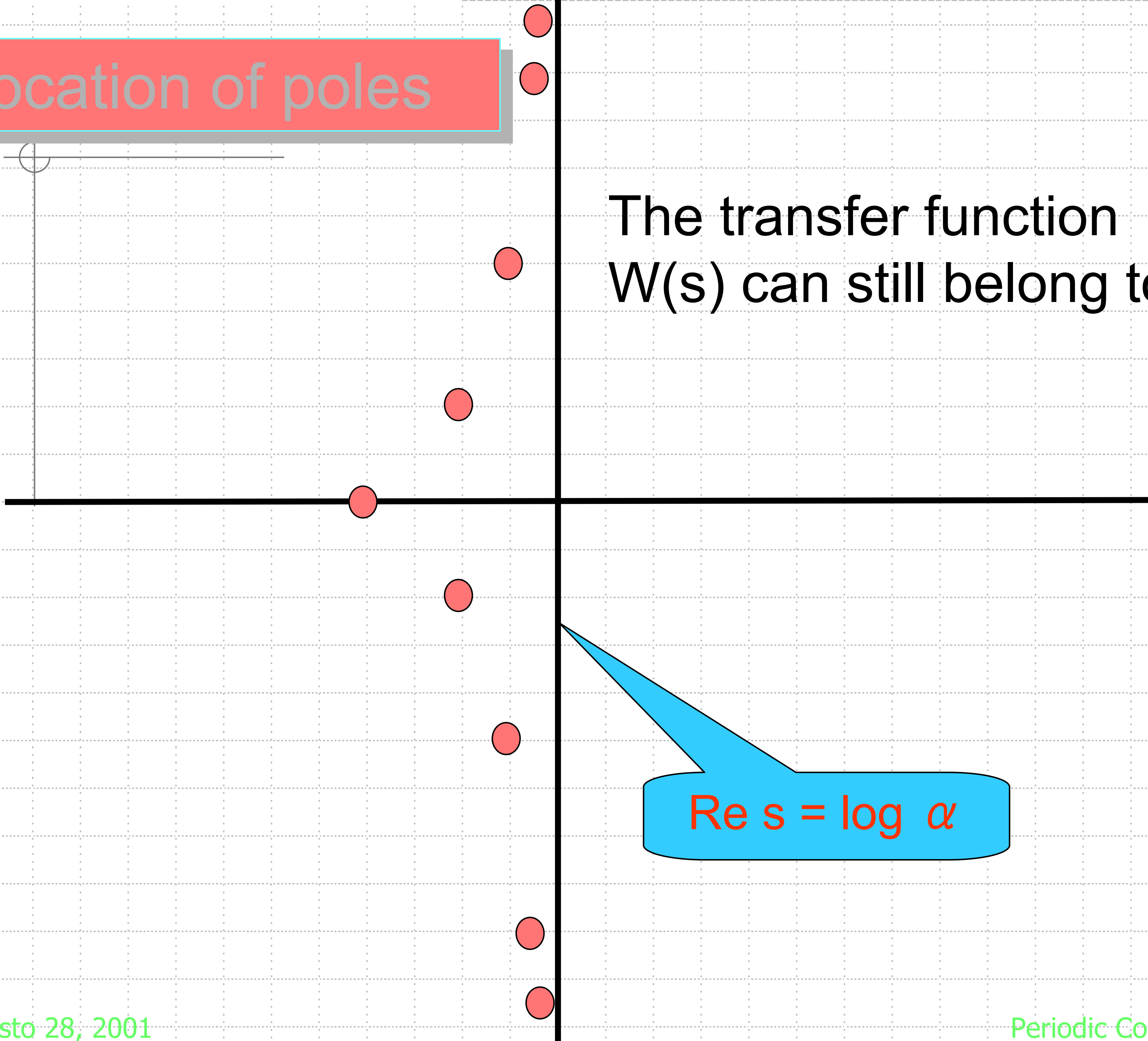
Stability Problems



- Neutral delay differential system
- The poles asymptotically approaches the axis Re s = log α, irrespective of P unless it has a feedthrough term
- Impossible to stabilize exponentially unless P is biproper

Location of poles

The transfer function W(s) can still belong to H^∞



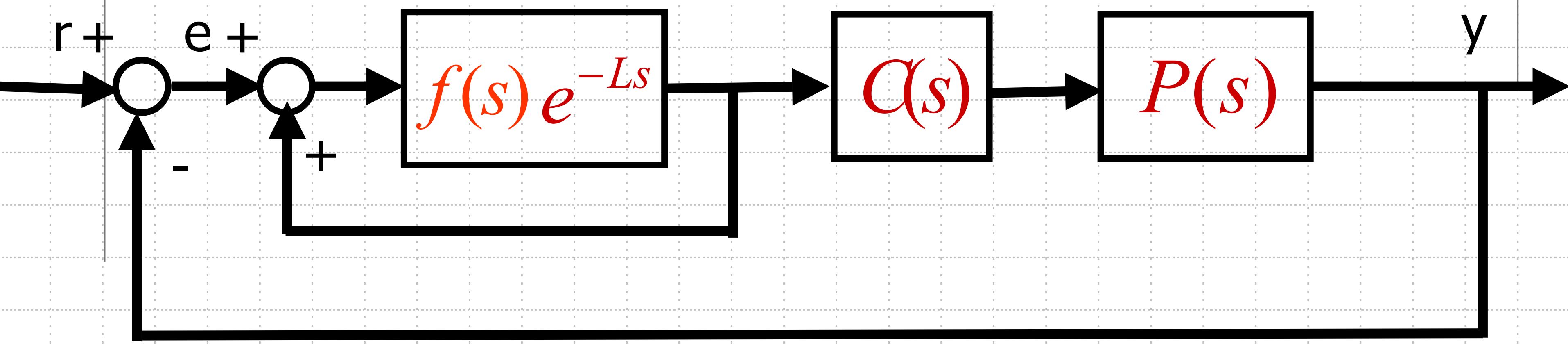
Difficulty in Stability/Stabilizability

- W(s) (closed-loop transf. fcn.) has infinitely many poles approaching Re s = 0 (neutral d-d systems; Hale)
- But W(s) can still belong to H^∞ (Logemann, SCL 88)
- Thus L^2 stable but not exp. stable
- Exp. stability ⇔ poles ⊂ {Re s ≦ -c < 0}
- Can never be achieved for strictly proper plant

Remedy

1. Introduce a low-pass filter into the delay
   - Exact internal model is lost
   - Problems in high-freq. tracking
2. Make it a discrete-time system
   - Lots of confusion (to be discussed later)

Modified Repetitive Control System



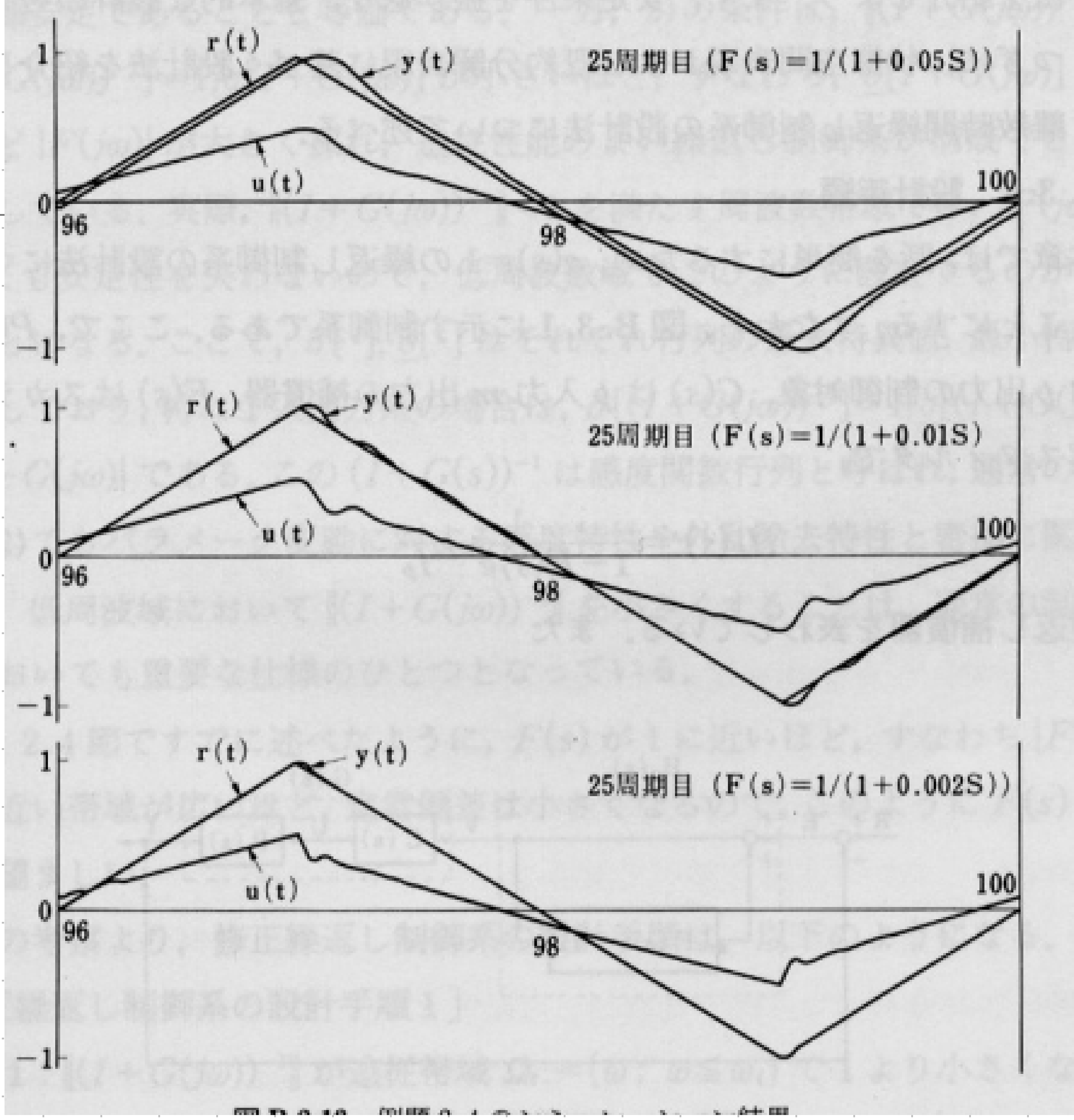
$f(s)$: low-pass filter

- $f(s)$ makes this system a "retarded" system; poles escapes away from the imaginary axis

Stability Condition

$$\left\| f(s)(1 - C(s)P(s)) \right\|_\infty < 1: \ H^\infty \text{ 1-block problem}$$

- Delay independent condition (thus finite-dimensional)
- Infinite-dim. Design:
  - Perry & Ozbay (ASME 97), Weiss (MTNS96)



Responses of a modified repetitive control system against 3 different filters; The same plant as before

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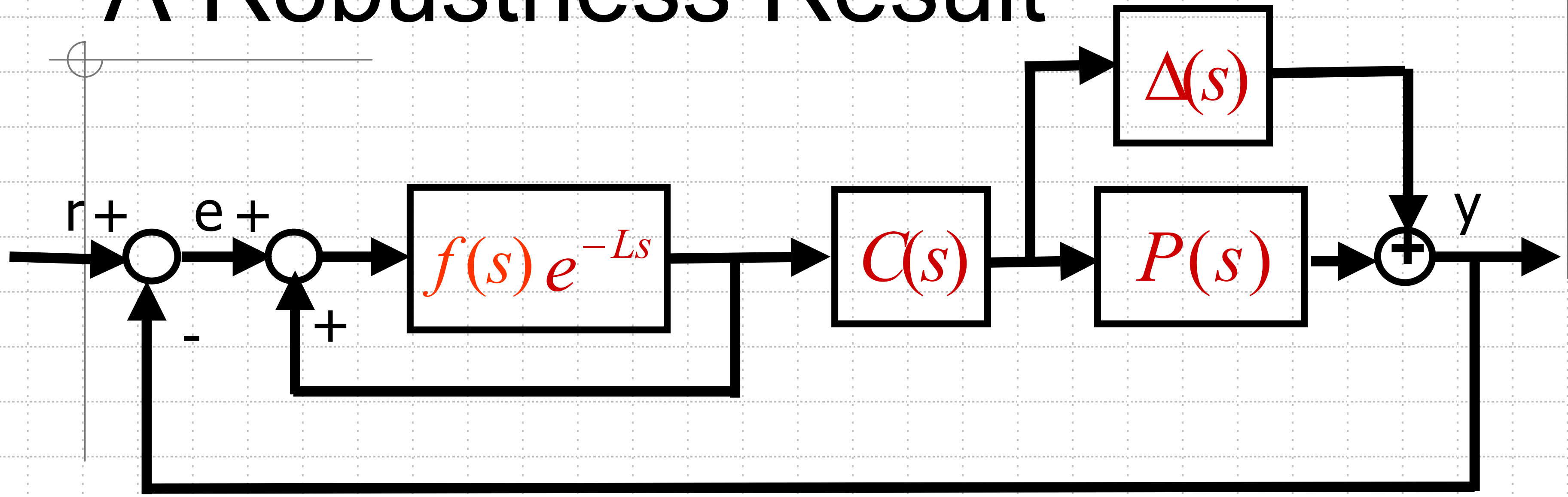
A Robustness Result



Suppose $\begin{cases} \left\| f(I-PC) \right\|_\infty = \gamma < 1 \\ \left|\Delta(j\omega)\right| < \left|r(j\omega)\right|, r \in H^\infty \end{cases}$

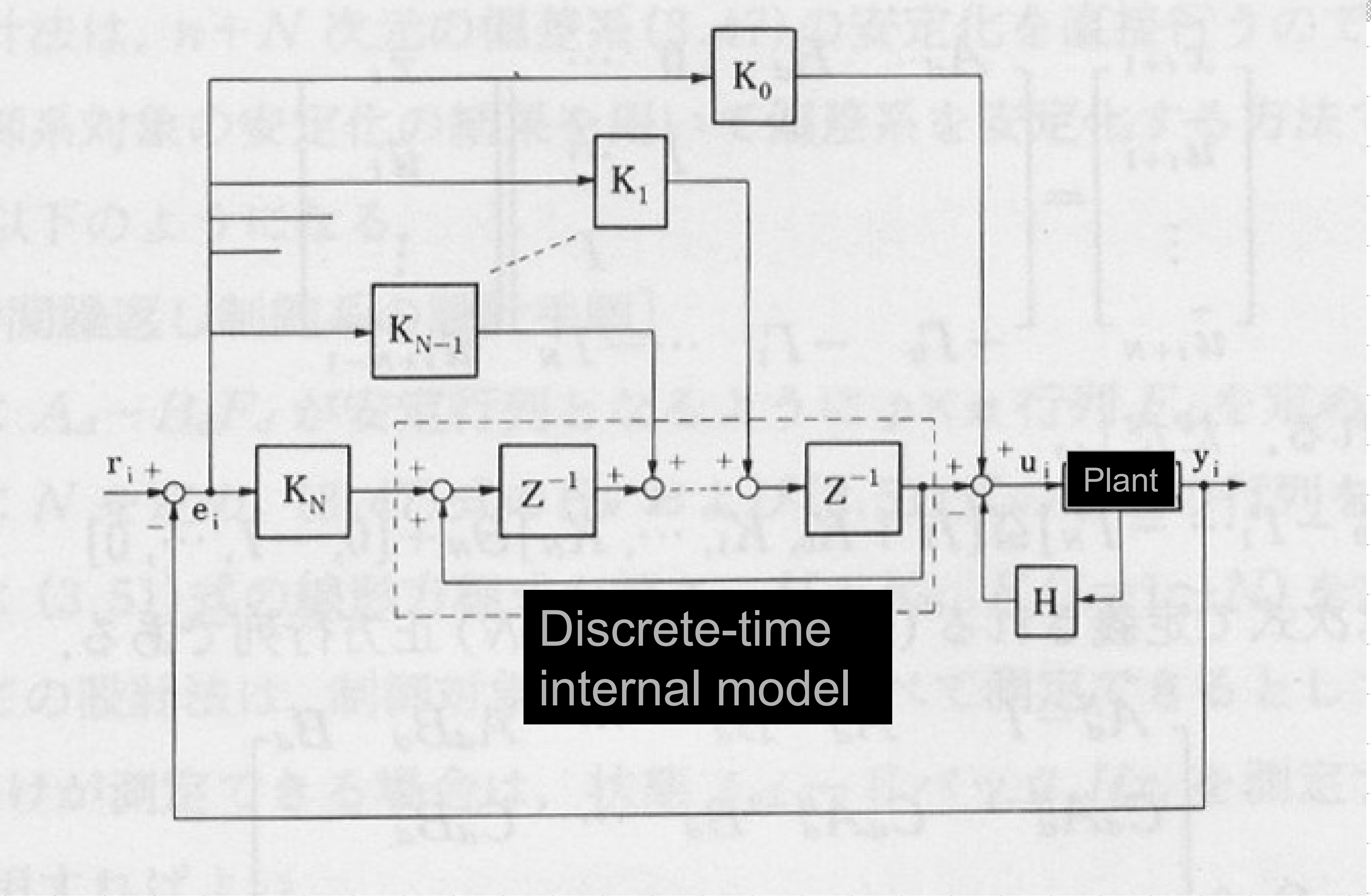
If $\left\| rfC \right\|_\infty < 1-\gamma,$ then this is robustly stable

To Summarize

- Precise tracking to all periodic signals ⇒ $1/(e^{Ls} - 1)$ is mandatory (Yamamoto-Hara TAC88)
- Rel. deg ≧ 1 ⇒ difficulty in stabilizability
  - ⇒ no exact repetitive control
- ⇒ modified repetitive control
  - H^∞ model matching problem (finite-dim.)
  - Infinite-dim. design is also attempted (Perry and Ozbay Trans ASME 97 ;Weiss MTNS96)
- Difficulty arising from ∞-dimensionality

Discrete-time counterpart

- Trade-off between stabilizability and tracking.
- Make it digital (Tomizuka et al. ASME89, and others):
  - No problems for stabilizability (aside from the obvious requirements)
  - In particular, no problem (at least superficially) in the relative degree of the plant if we allow delayed tracking
  - Can lose trackability in the intersample behavior
  - Needs some framework to assure tame intersampling behavior
- Lots of historical confusion (and still are).



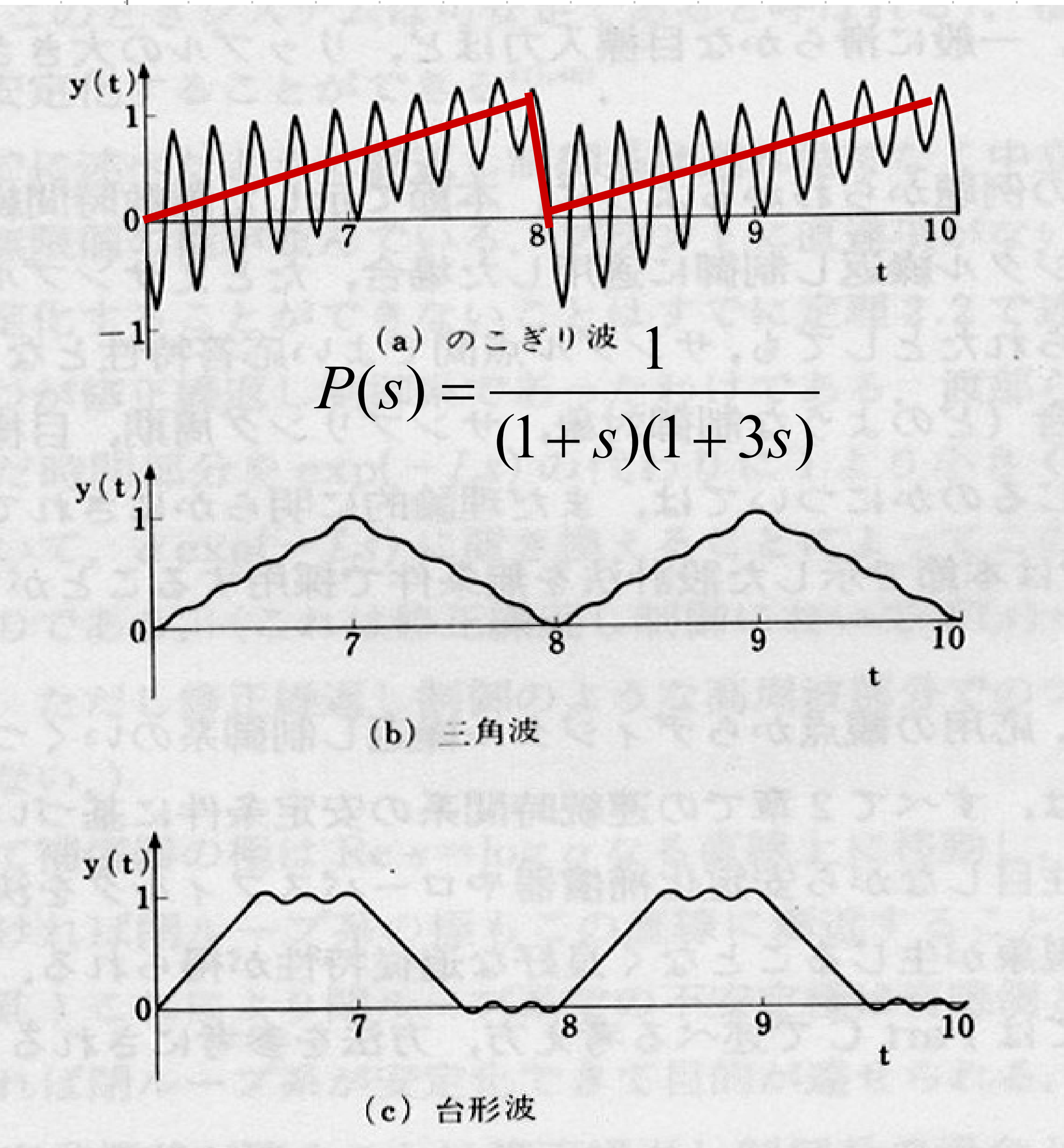
Digital Repetitive Control System

Confusion

- The worst one:
  - Everything is resolved by going over to discrete-time: no problems in stability and tracking
  - Much confusion in many submitted articles
- Facts:
  - Reference signals are cont.-time; Tracking achievable only at sampled points: No capability for tracking in the intersample
  - Often very large intersample ripples (Hara, Kondo, etc., CDC '90) Numerically fragile
  - Needs a framework for discussing intersample ripples and attenuates the high-freq. components

Ripples in Digital Repetitive Control

Hara, Kondo, etc., CDC '90



(a) のこぎり波

$$P(s) = \frac{1}{(1+s)(1+3s)}$$

(b) 三角波

(c) 台形波

Tracking is achieved at sampled points
But it can exhibit large intersample ripples

Why?

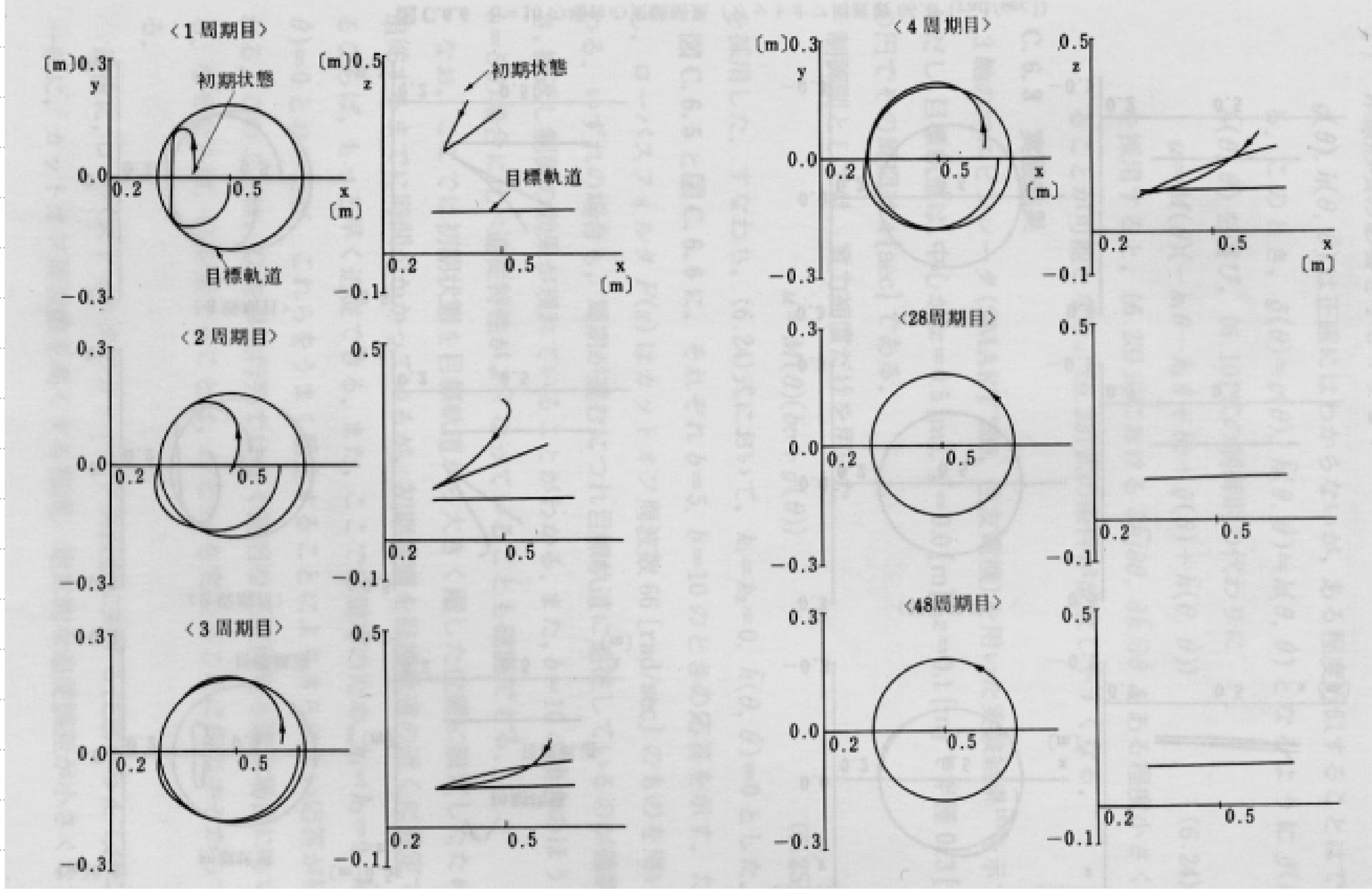
- Not quite understood as yet (not much follow-up study, except by Hara et al. ACC92)
- However, note that
  - Relative degree = 2
  - not stabilizable in the cont.-time
- *What cannot be achieved in the continuous-time case can be achieved in digital?*

Remedy?

- This problem is not well explored in the literature (except one follow-up by Hara)
- The mainstream of study is focused around digital repetitive control without much attention on the intersample behavior
- Note: Just filtering out the reference signal may not be enough (previous example)

Nonlinear Repetitive Control

- Omata, Hara & Nakano J. Robotic Syst.87; passivity theory
- Ghosh and Paden TAC00; some extensions
- Lucibello CDC93; new internal model principle(?)



Nonlinear Repetitive Control for a robotic motion
By Omata, Hara and Nakano (87)

Some Remaining Issues

- For LPTV systems: Sison & Chong (CDC97) …
- Many search algorithms for (nonlinear) ILC: Driessen, Sadegh, et al. (CDC98)
- Internal model principle: de Roover and Bosgra (ACC97, CDC97); purely discrete-time; no intersample consideration; unification of repetitive and ILC
- Robustness issues: Yamamoto & Hara (Automatica 92), Lee & Smith (CDC96); Weiss (MTNS96; Automatica)
- Generalization to multiple periods: Chang & Suh (CDC96), Weiss,
- Applications to mechanical systems: Tomizuka, Sadegh and others

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ILC (Iterative Learning Control)

- Arimoto, Miyazaki, Kawamura and others
- Some ignored relationships with repetitive control
- Actually based on almost the same idea

Basic Idea

$$\Sigma: \begin{cases} \dot{x} = f(x) + Bu \\ y = Cx \end{cases}$$

Objective: Given $r \in L^2[0,T]$,
find u_{opt} such that $y = r$

- Finite time problem; initial state x_0 is given
- Initialize appropriately
- Repeat
  - $u_{k+1} = u_k + \Gamma d(e_k)/dt$
    under resetting of initial states

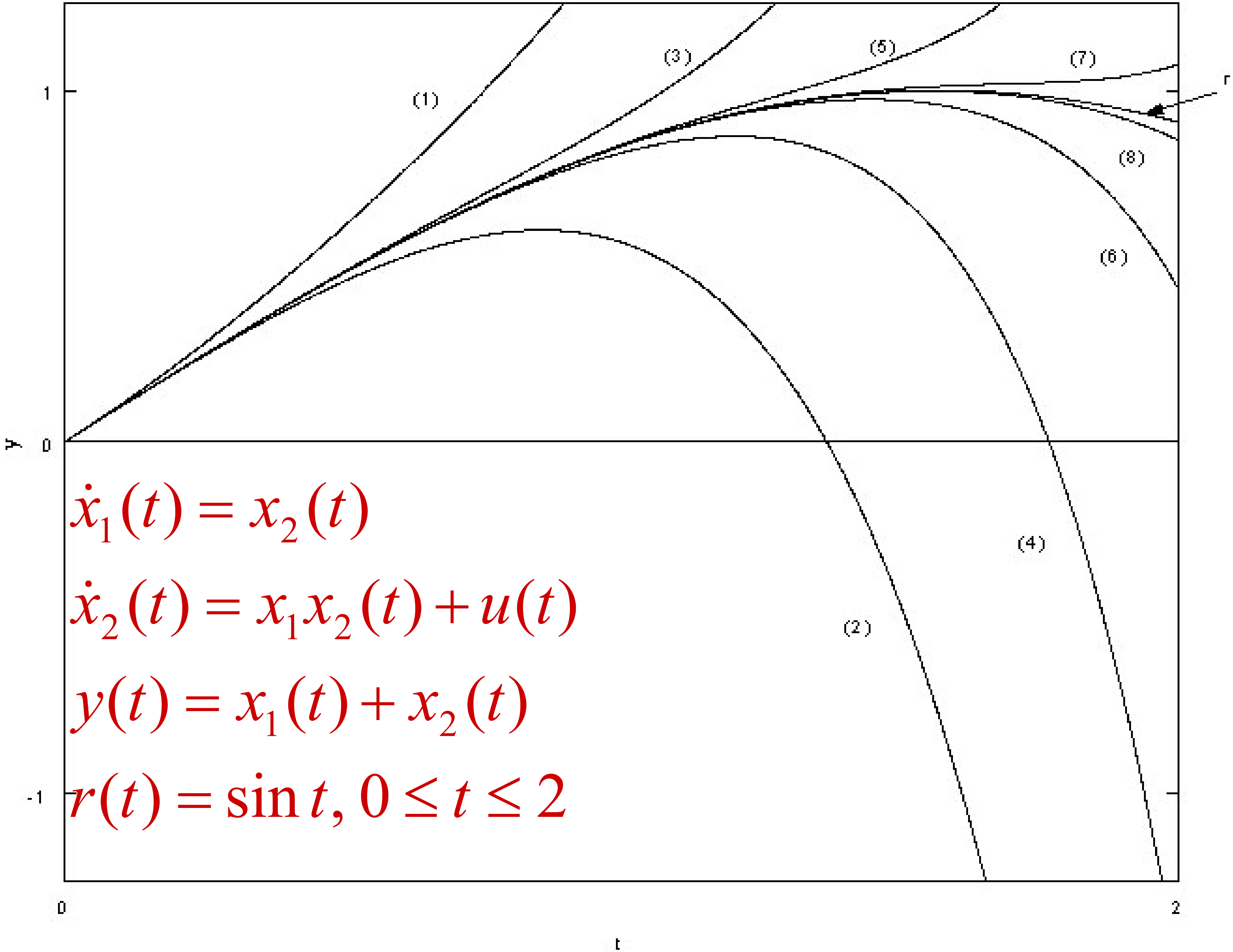
Convergence Condition

$$\|I - CB\Gamma\| < 1 \text{ (Arimoto et al.)}$$

- Observation:
  - CB must be full rank
  - Very close to the repetitive control small gain condition
- The trick:
  - Relative degree is 1 (coefficient: CB)
  - Main term is CB
  - To make the feedthrough term, introduce the derivative of e_k
  - The rest can be estimated by the Gronwall inequality using the finite-time tracking property

Relationships

- Hence close to $\|I - PC\| < 1$ (repetitive control stability condition)
- The difference is the finite-time property (hence only CB is in the condition)
- Differentiation → feedthrough term
- Stability is of less importance (finite-time tracking) → applicable to an unstable systems



$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = x_1 x_2(t) + u(t)$$

$$y(t) = x_1(t) + x_2(t)$$

$$r(t) = \sin t,\ 0 \le t \le 2$$

Tracking to sin t via ILC

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Some Current Issues in ILC

- Adaptive ILC (Rogers & Owens)
- ILC for nonminimum phase plants
  - Basically not a problem; but nonminimum phase case tends to produce large inputs
- Relationships with repetitive control
- Nonlinear ILC

To Summarize

- Difficulty: trade-off between stability (error convergence for ILC) vs. high-freq. tracking
- Need not be easily compromised
- Low-pass filter → modified rep. control
- Or ZPETC (Tomizuka ACC88) – dicrete-time variant
- Discrete-time rep. control: regular finite-dim. system
  - Not difficult if intersample behavior is not taken into account
  - If the intersample behavior (high-freq. performance) is considered, it requires the modern sampled-data theory
  - Not much has been done in this direction: Langari-Francis (ACC94), Ishii-Yamamoto (CDC98)

Some Future Issues

- Study by the sampled-data theory; note: Good sample-points behavior need not mean good performance in intersample behavior
- Further robustness studies
- More continuous-time analysis
- Multirate study
  - To supplement the intersample behavior
  - Multi-period repetitive control

Thank you for your attention

Literature

- Robustness studies
  - Yamamoto & Hara Automatica 92; Lee & Smith CDC96; WeissMTNS96
- Nonlinear repetitive control
  - Omata, Hara & Nakano J. Robotic Systems 87; Ghosh & Paden TAC00;
- Digital repetitive control
  - Hu & Tomizuka ASME 94,
  - Some discrete-time design
    - Fabian ACC99; Kim & Tsao ACC01;
  - Parameter space design
    - Guvenc ACC 01;
- Ripple Analysis and Attenuation
  - Hara, Tezuka & Kondo CDC00; Hara Kawamura & Sung ACC92
- Sampled-data design
  - Langari & Francis ACC94; Ishii & Yamamoto CDC98
- Adaptive repetitive control
  - Tsao & Tomizuka ASME94,

Continued

- Discrete-time internal model principle
  - De Roover & Bosgra CDC97
- Applications
  - Yau & Tsai (Moter control) ACC99; Zhou, Wang & Xu ACC00; Zhou and Wang CDC00
- Other Design Methods
  - Chen, Longman CDC99; Kondo et al. CDC97, Sison & Chong and many others
  - Chen & Longman (smooth updates) CDC99 strange reasoning for the small gain condition $\| I-\Gamma G \| <1$
  - Koroglu & Morgul (LQ design) ACC99
- LPTV systems
  - Sison & Chong CDC97
- Dual rate problem
  - Chang & Suh CDC96, Yamada et al. CDC00

Continued

- ILC (many others)
  - K. L. Moore(Book; CDC99)
  - Driessen, Sadegh and Kwok (Line search) CDC98
  - Adaptive
    - French, Munde, Rogers and Owens CDC99

Comments

- ZPETC (Tomizuka and others; Zero Phase Error Tracking Control) introduces a low-pass filter to take care of high frequencies (similar to spectral factorization); high-freq. roll-off to take care of robust stability
- Others often ignore this. Just some ways of stabilizing a special discrete-time system